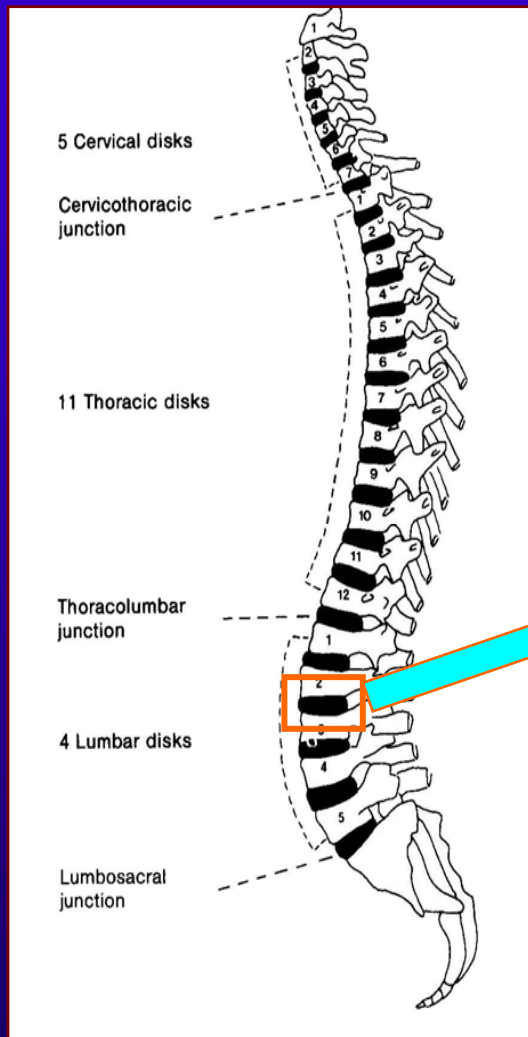


Multiphasic Mechano-Electrochemical Finite Element Model of Human Intervertebral Disc

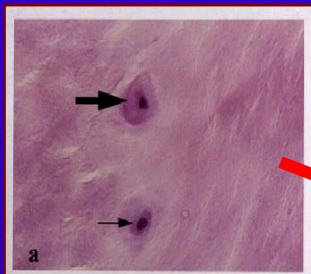
Hai Yao, PhD

Clemson-MUSC Bioengineering Program
Department of Bioengineering
Clemson University, Clemson, SC

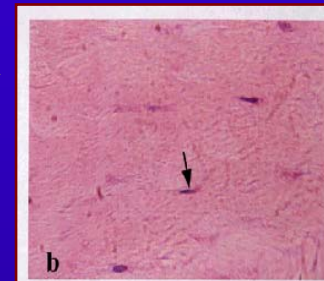
Spine and Intervertebral Disc (IVD)



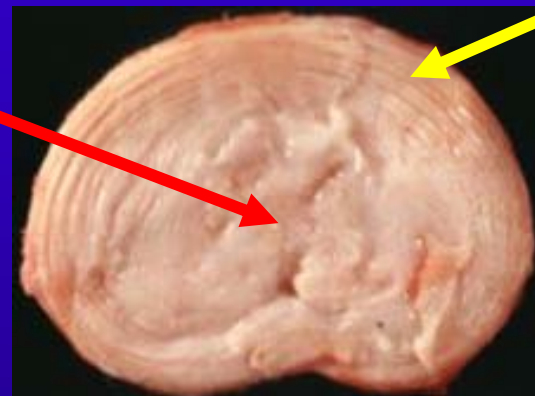
Nucleus Pulposus



Annulus Fibrosus



Adult Human Disc



Composition	IVD	Annulus Fibrosus	Nucleus Pulposus
Water (% wet wt.)	65-90%	60-70%	70-90%
Collagen (% dry wt.)	10-60%	50-70%	15-25%
Proteoglycans (% dry wt.)	15-65%	10-20%	~50%

(adapted from Kramer, 1981)

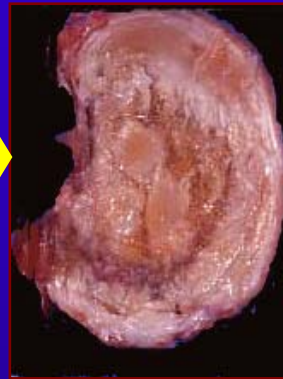
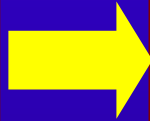
(Mow et al., 1990)

Disc Degeneration

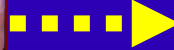
● Disc degeneration



Normal



Degenerated



Low Back Pain

- Discomfort in lumbar spine
- 11.7 million people (5.2%) /year
- \$20 billion expenditure /year

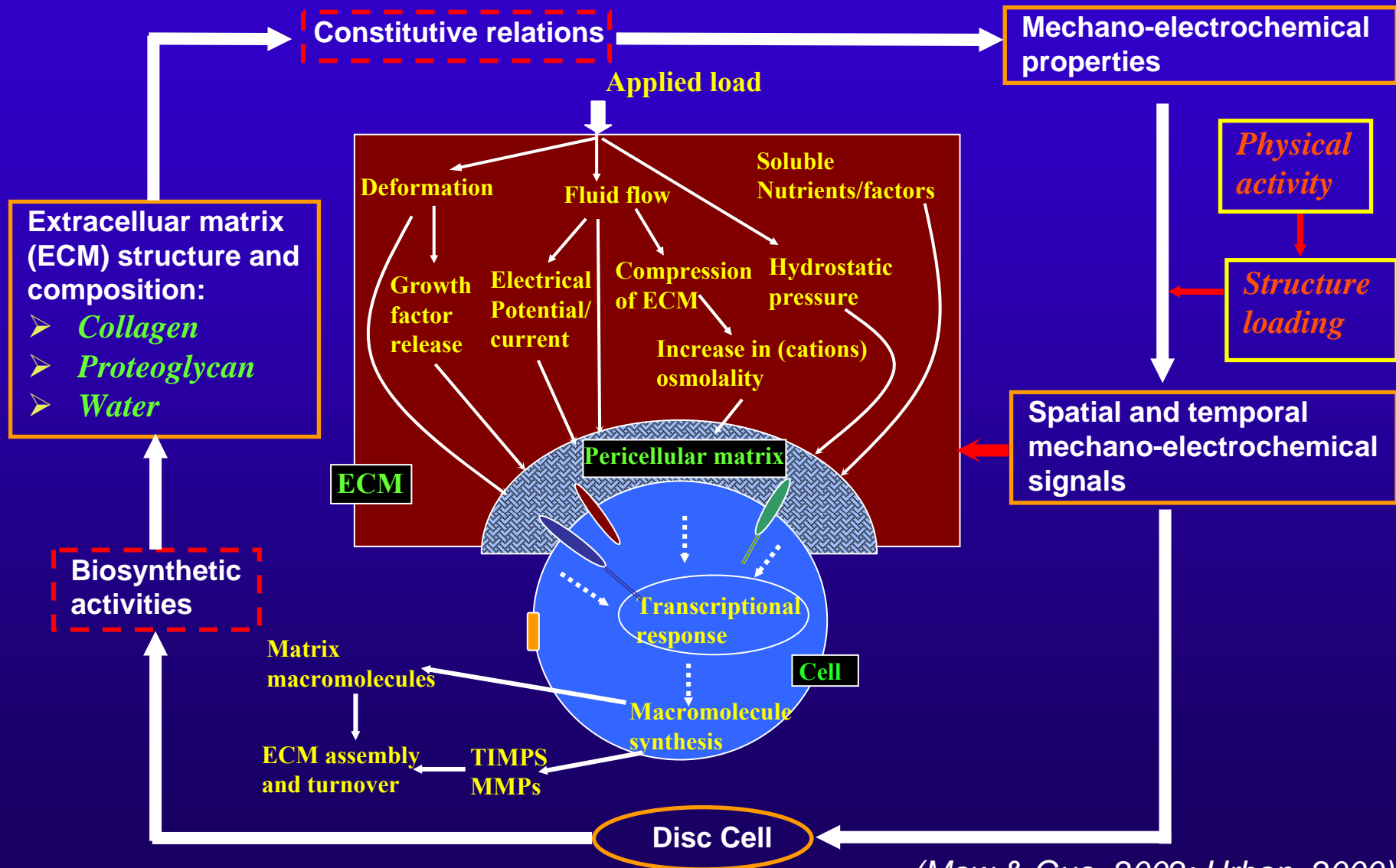
❑ Failure of disc cells to balance matrix synthesis with degradation

❑ Possible mechanism

- Failure of nutrient supply
- Mechanical load and injury
- Genetic factors

(Urban & Roberts, 2003)

Inter-relationships between ECM, Signals and Cell



(Mow & Guo, 2002; Urban, 2000)

Disc Modeling

- **Viscoelastic single phase model : solid**

Mechanical signals (*strain & stress*)

(Belytschko et al., 1974; Shirazi-Adl et al., 1986)

- **Biphasic (poroelastic) theory: solid + fluid**

Mechanical signals (*strain, stress, fluid pressure & flux*)

(Laible et al., 1993; Simon et al., 1996; Whyne et al., 2001)

- **Multiphasic theory: solid + fluid + ions + neutral solute**

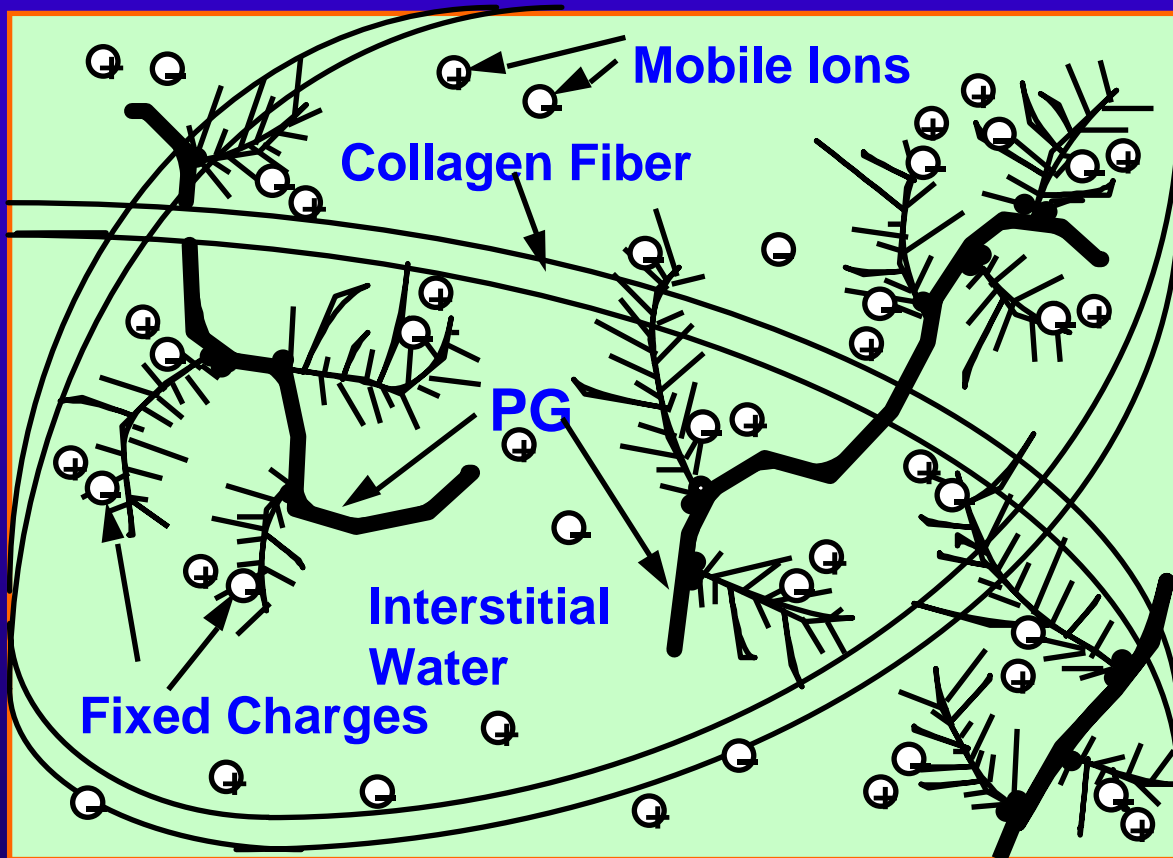
Mechano-electrochemical signals (*ion and/or solute concentration*)

- **Solid+fluid+ions (no neutral solutes)** (*Frijns et al., 1997; Iatridis et al., 2003*)
- **Solid+fluid+neutral solute (no ions)** (*Mauck et al., 2003; Ferguson et al., 2004*)
- **Homogenous FE model with all species** (*Yao & Gu, 2004, 2006*)

Objectives

- **Develop 3D inhomogeneous Finite Element Model (FEM) for human disc based on the mechano-electrochemical mixture theory with multi-species**
- **Quantify the physical signals and the transport of fluid, ions and neutral solutes within the disc during compressive stress relaxation**
 - **Effects of tissue composition and mechanical loading on physical signals and solute transport**

Extracellular Matrix of Charged Hydrated Soft Tissue



Multiphasic Feature:

- Solid phase (charged)
- Interstitial water phase
- Ion phases (m species)
- Neutral solute phases (n species)

Mechano-Electrochemical Mixture Theory *(Lai, et al. 1991)*

Disc tissue:

Isotropic inhomogeneous mixture

Elastic solid + Fixed charge
Water
Ions
Neutral solute

● Governing equations

□ Balance of linear momentum (quasi-static)

(Lai et al, 1993; Gu et al, 1998; Sun et al, 1999)

Tissue $\nabla \cdot \boldsymbol{\sigma} = 0$

Water $\mathbf{J}^w = -RTk \left(\nabla \varepsilon^w + \frac{c^+}{\varepsilon^+} H^+ \nabla \varepsilon^+ + \frac{c^-}{\varepsilon^-} H^- \nabla \varepsilon^- + \frac{c^o}{\varepsilon^o} H^o \nabla \varepsilon^o \right)$ $\boldsymbol{\sigma}$: tissue stress
 \mathbf{J}^α : flux relative to solid

Cation $\mathbf{J}^+ = H^+ c^+ \mathbf{J}^w - \frac{\phi^w c^+ \mathbf{D}^+}{\varepsilon^+} \nabla \varepsilon^+$ ϕ^w : water content c^α : concentration
 k : permeability ε^α : chemical potential

Anion $\mathbf{J}^- = H^- c^- \mathbf{J}^w - \frac{\phi^w c^- \mathbf{D}^-}{\varepsilon^-} \nabla \varepsilon^-$ \mathbf{R} : gas constant \mathbf{D}^α : diffusivity
 \mathbf{T} : temperature \mathbf{H}^α : hindrance factor

Neutral solute $\mathbf{J}^o = H^o c^o \mathbf{J}^w - \frac{\phi^w c^o \mathbf{D}^o}{\varepsilon^o} \nabla \varepsilon^o$ $\alpha = w$ (water), $+$ (cation), $-$ (anion), o (solute)

Theory (cont.)

Governing equations (cont')

Balance of mass and charge

$$\mathbf{v}^s = \frac{\partial \mathbf{u}}{\partial t} \longrightarrow \nabla \cdot (\mathbf{v}^s + \mathbf{J}^w) = 0 \text{ Tissue}$$

$$\phi^w = 1 - \phi^s = \frac{\phi_o^w + e}{1 + e} \text{ Water}$$

$$\partial(\phi^w c^+) / \partial t + \nabla \cdot (\mathbf{J}^+ + \phi^w c^+ \mathbf{v}^s) = 0 \text{ Cation}$$

$$\partial(\phi^w c^-) / \partial t + \nabla \cdot (\mathbf{J}^- + \phi^w c^- \mathbf{v}^s) = 0 \text{ Anion}$$

$$\partial(\phi^w c^o) / \partial t + \nabla \cdot (\mathbf{J}^o + \phi^w c^o \mathbf{v}^s) = 0 \text{ Neutral solute}$$

Constitutive relations

$$\boldsymbol{\sigma} = - \left[RT \boldsymbol{\varepsilon}^w + RT \phi (c^+ + c^- + c^o) - p_o \right] + \lambda \nabla \cdot \mathbf{u} + \mu \left[\nabla \mathbf{u} + (\nabla \mathbf{u})^T \right]$$

$$\boldsymbol{\varepsilon}^w = \frac{p}{RT} - \phi (c^+ + c^- + c^o)$$

$$\boldsymbol{\varepsilon}^+ = \gamma_+ c^+ \exp\left(\frac{F_c \psi}{RT}\right)$$

$$\boldsymbol{\varepsilon}^- = \gamma_- c^- \exp\left(-\frac{F_c \psi}{RT}\right)$$

$$\boldsymbol{\varepsilon}^o = \gamma_o c^o$$

Fixed charge $c^F = \frac{c_0^F (1 - \phi^w) \phi_0^w}{(1 - \phi_0^w) \phi^w}$

$$k = a \left(\frac{\phi^w}{\phi^s} \right)^n$$

Permeability
(Yao & Gu, 2003)

$$\frac{D}{D_0} = \exp \left[-\alpha \left(\frac{r_s}{\sqrt{\kappa}} \right)^\beta \right]$$

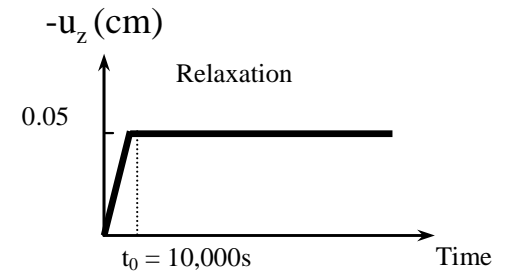
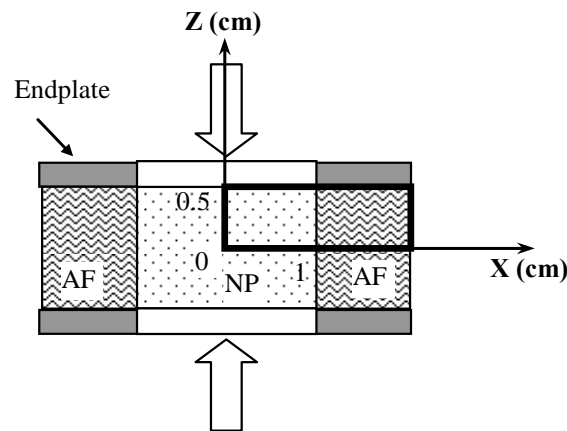
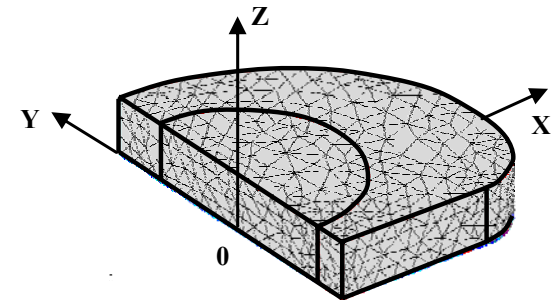
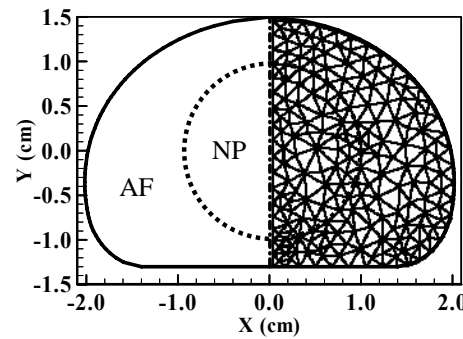
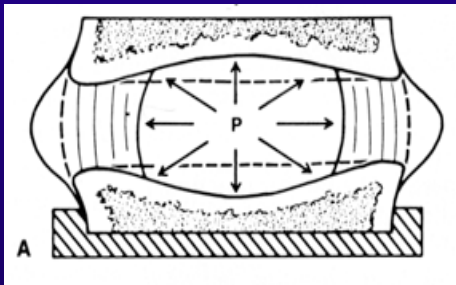
Diffusivity
(Gu & Yao, 2004)

Geometry, Mesh and Load

Adult human IVD



$u_z(t)$



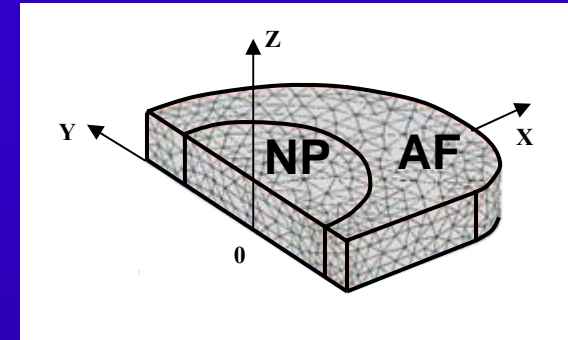
Displacement
at $z=0.5$ cm

Initial & Boundary Conditions

Initial condition

$$t = 0 \quad \mathbf{u} = 0, \quad \varepsilon^w = \varepsilon^{w*}, \quad \varepsilon^+ = \varepsilon^{+*}, \quad \varepsilon^- = \varepsilon^{-*}, \quad \varepsilon^o = 0$$

(* The quantities in the bathing solution)



Boundary condition (adhesive platens)

$$\text{Plane } x = 0 \quad u_x = \sigma_{xy} = \sigma_{xz} = 0, \quad J_x^w = J_x^+ = J_x^- = J_x^o = 0$$

$$\text{Plane } z = 0 \quad u_z = \sigma_{zx} = \sigma_{zy} = 0, \quad J_z^w = J_z^+ = J_z^- = J_z^o = 0$$

$$\text{Plane } z = 0.5 \quad u_z = -u_z(t), \quad u_x = u_y = 0$$

Permeable:

$$\varepsilon^w = \varepsilon^{w*}, \quad \varepsilon^+ = \varepsilon^{+*}, \quad \varepsilon^- = \varepsilon^{-*}, \quad \varepsilon^o = \varepsilon^o H(t)$$

Impermeable:

$$J_z^w = J_z^+ = J_z^- = J_z^o = 0$$

Lateral surface

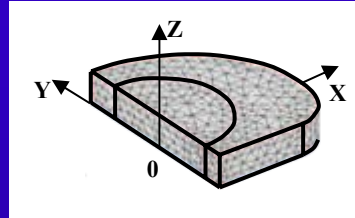
$$\boldsymbol{\sigma} \cdot \vec{n} = 0, \quad \varepsilon^w = \varepsilon^{w*}, \quad \varepsilon^+ = \varepsilon^{+*}, \quad \varepsilon^- = \varepsilon^{-*}, \quad \varepsilon^o = \varepsilon^o H(t)$$

H(t): Heaviside step function

Numerical Implementation

● Problem solving (*FEMLAB 3.1, COSMOL Inc.*)

□ Meshing



4306 second-order tetrahedral elements

□ Solver

- Based on implicit Euler backward scheme
- Variable scaling

□ Time step

- Ramp phase: 100s (maximum) ; Relaxation phase: 5s ~ 1000s

□ Convergence

- Error tolerance tightened
- Mesh refined

Numerical accuracy checked against the triphasic stress-relaxation result in literature

(Mow et al, 2002; Sun et al, 2004)

Parameters

	AF	NP
Initial water content ϕ_0^w	0.75	0.86 (0.82, low WC NP)
Initial fixed charge density c_0^F (mEq/ml)	0.15	0.25 (0.15, low FCD NP)
Elastic constant λ (MPa)	0.2	0.02 (0.002, soft NP)
Elastic constant μ (MPa)	0.15	0.015 (0.0015, soft NP)
Darcy permeability (nm ²) $\kappa = a_1 (\phi^w / \phi^s)^{b_1}$	$a_1 = 0.00044,$ $b_1 = 7.193$	$a_1 = 0.00339,$ $b_1 = 3.24$
Relative diffusivity $\frac{D^\alpha}{D_0^a} = \exp \left[- a_2 \left(\frac{r_s^a}{\sqrt{\kappa}} \right)^{b_2} \right]$	$a_2 = 1.29,$ $b_2 = 0.372$	$a_2 = 1.25,$ $b_2 = 0.681$

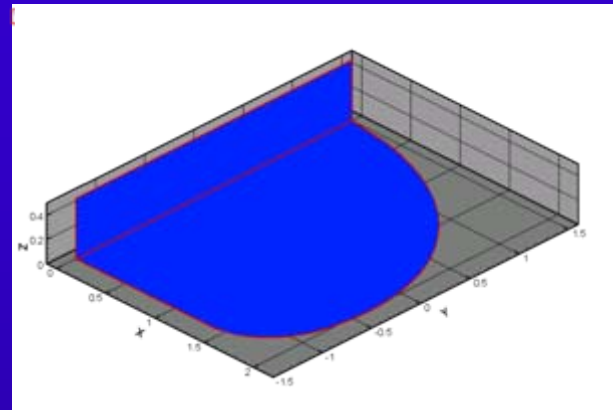
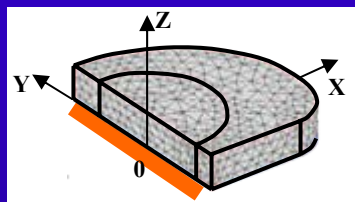
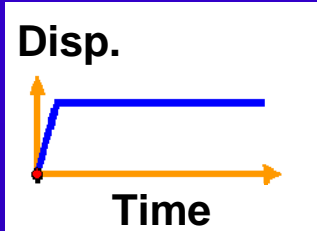
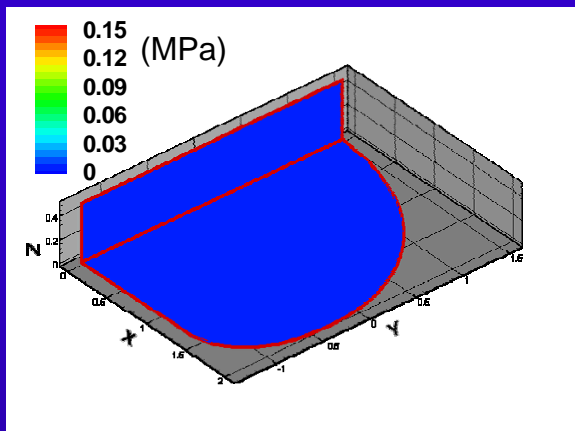
Ions: $c^* = 0.15$ M NaCl

$r_s^+ = 0.197$ nm, $r_s^- = 0.142$ nm

Neutral solute (IGF-1): $c^{0*} = 4 \times 10^{-5}$ mol/m³

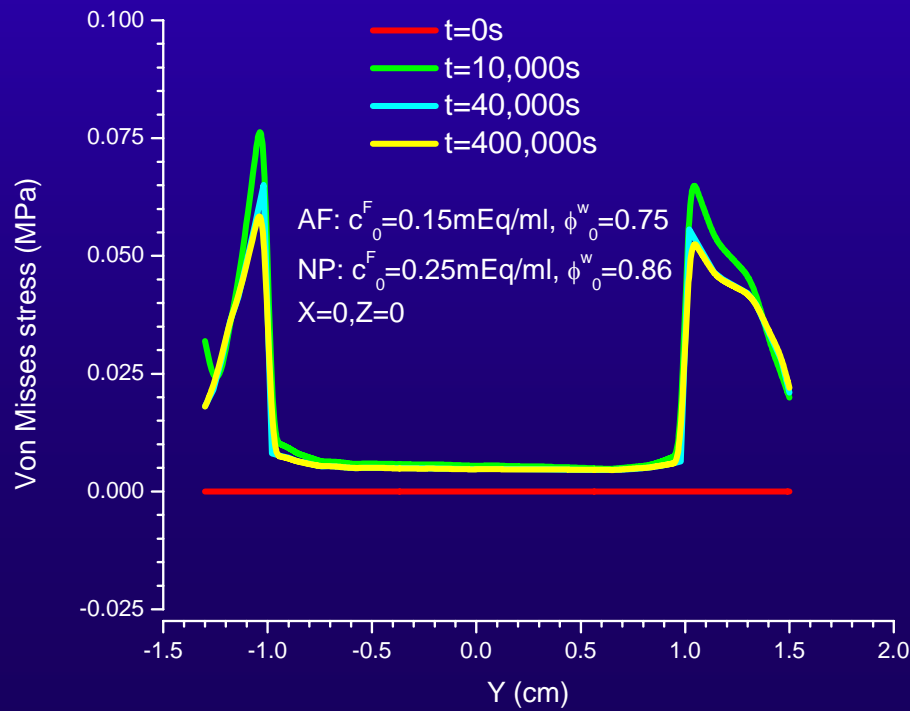
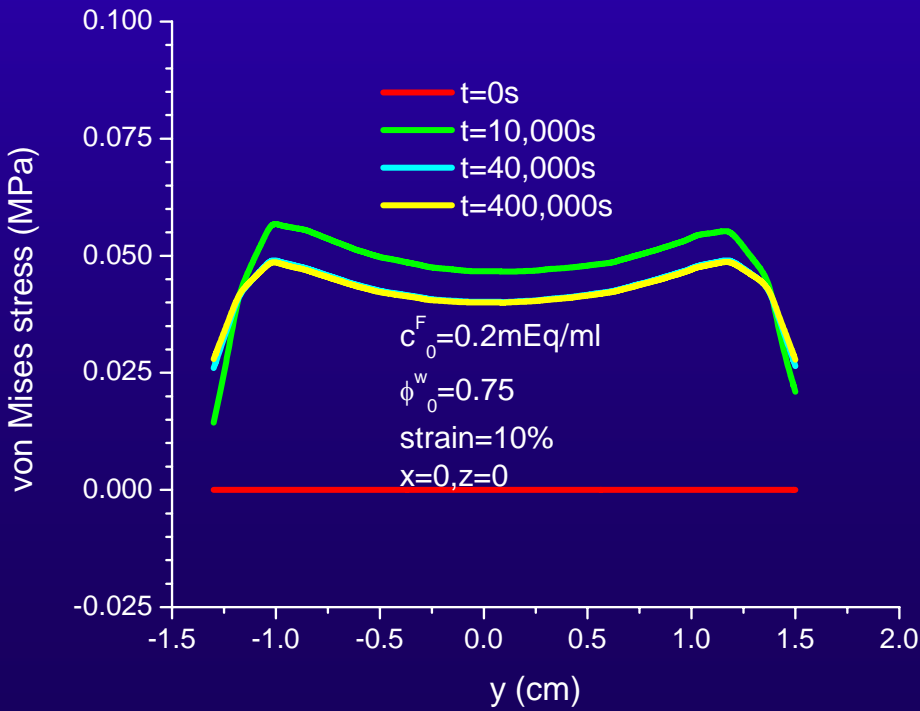
$r_s^0 = 1.146$ nm, $\Phi = 0.1$

Results: von Mises stress

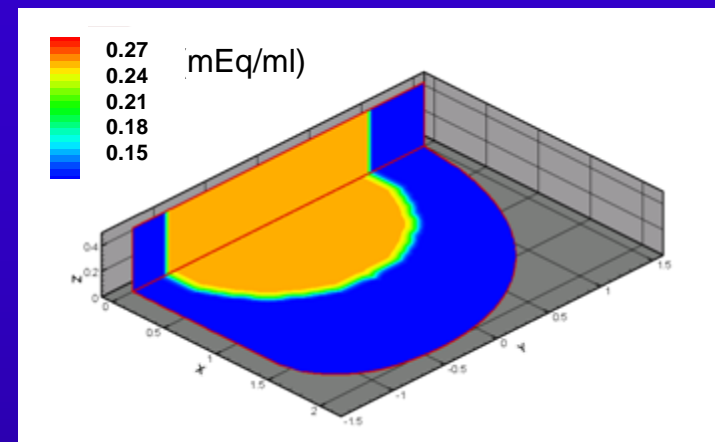
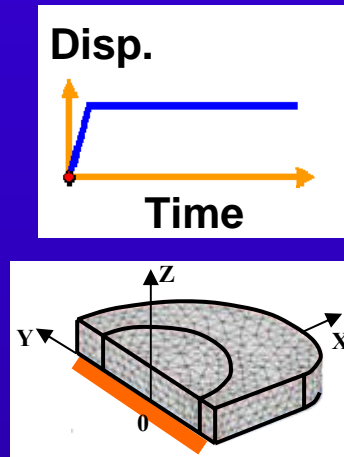
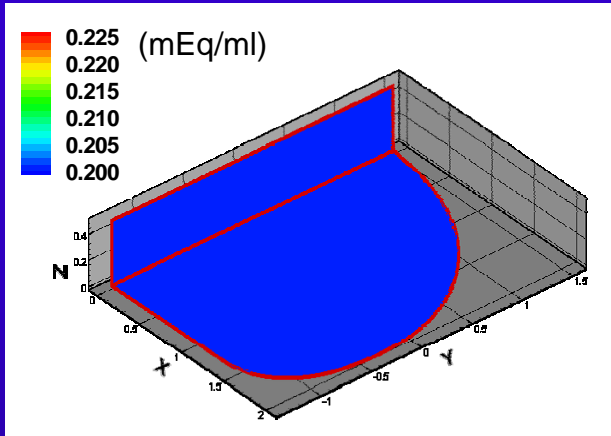


Homogeneous

Inhomogeneous

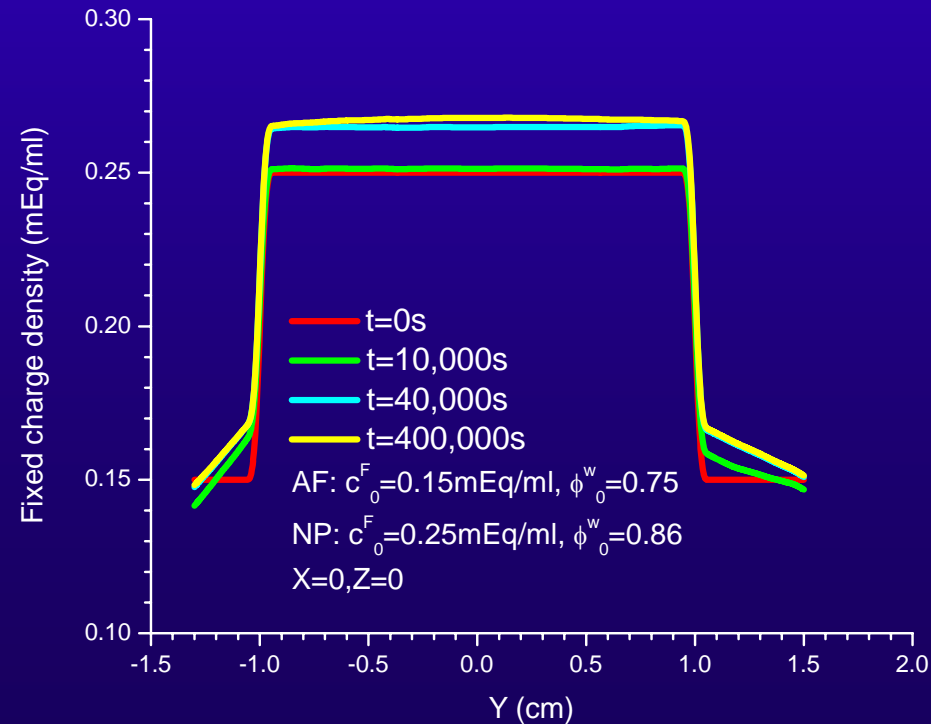
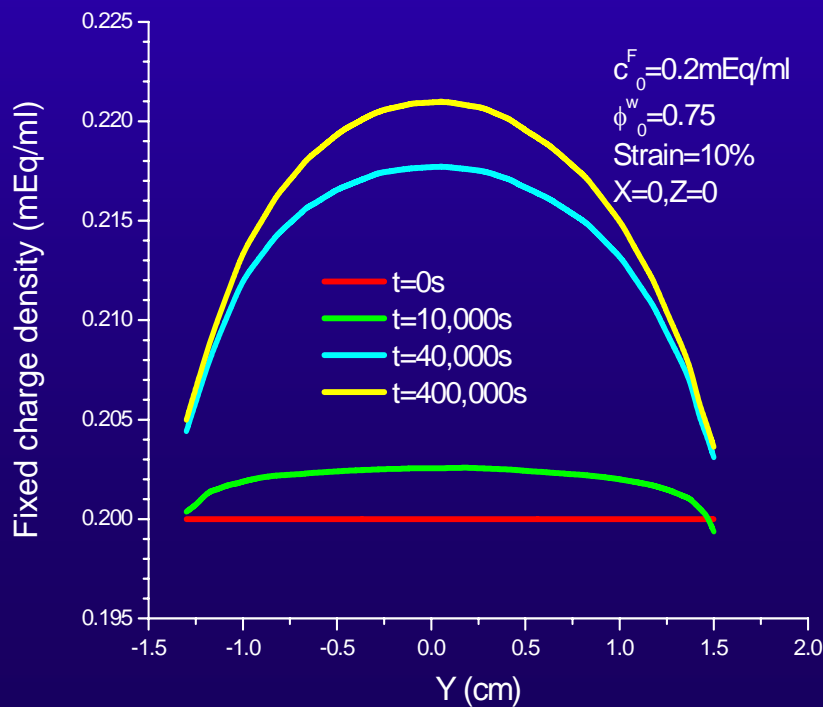


Fixed charge density (FCD)

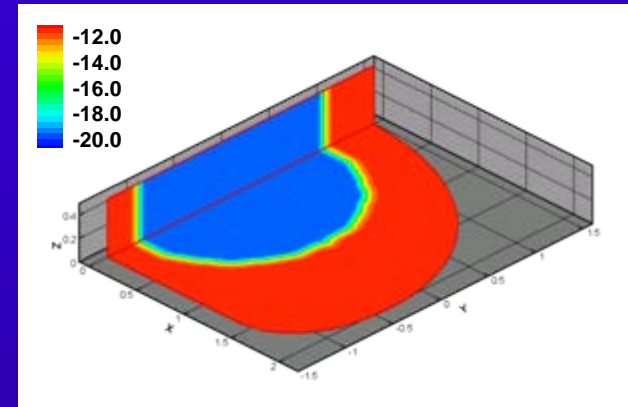
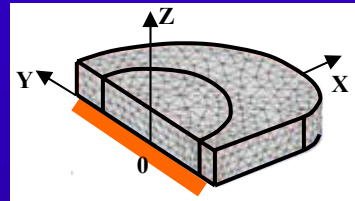
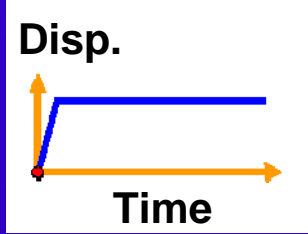
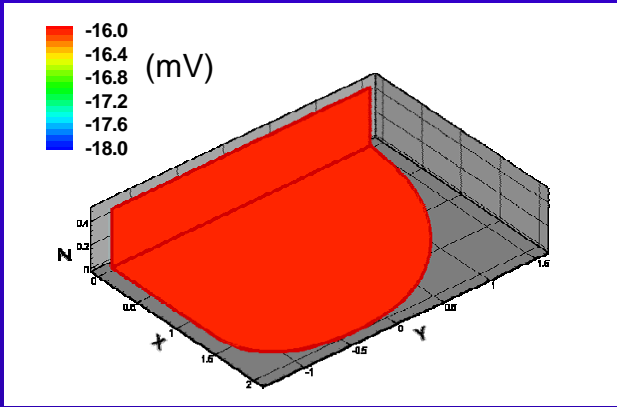


Homogeneous

Inhomogeneous

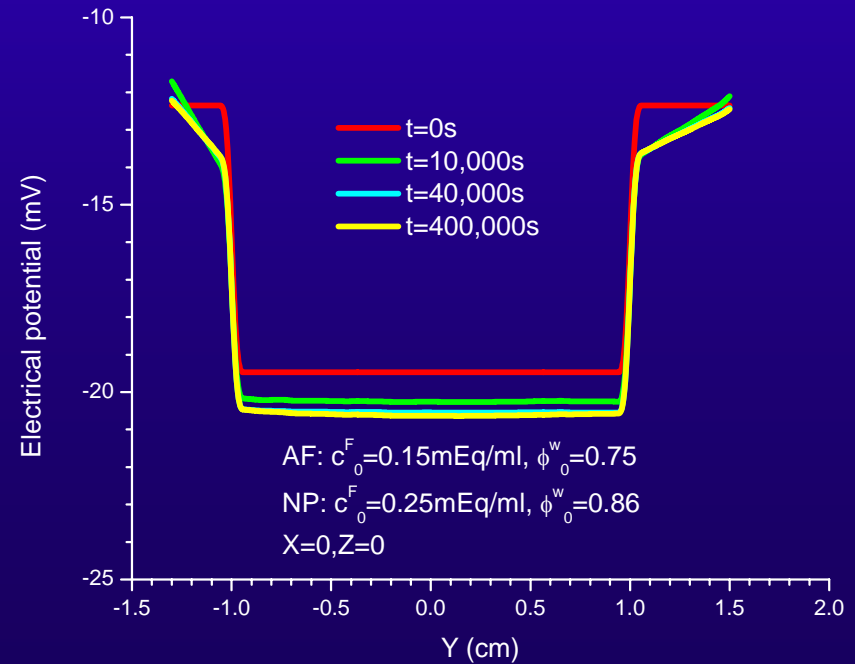
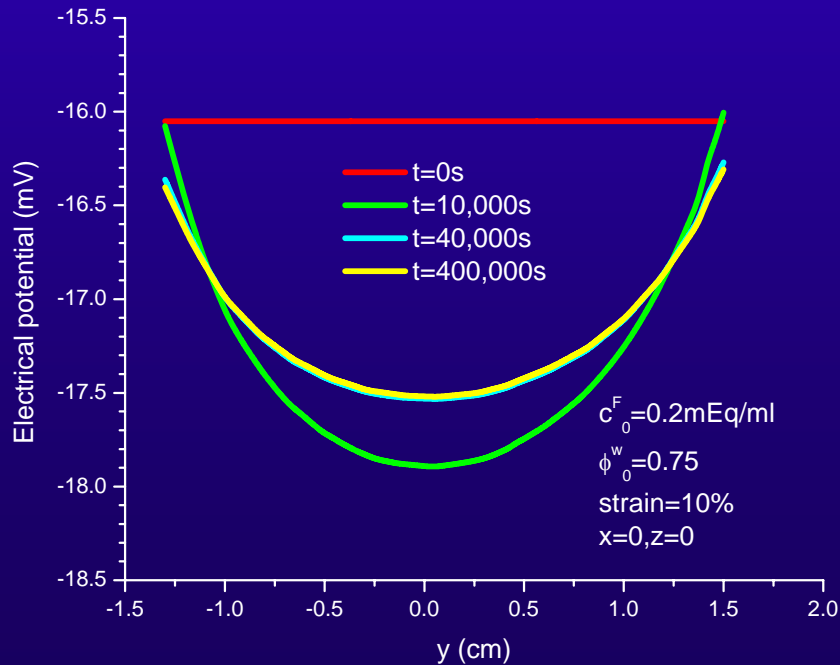


Electrical Potential

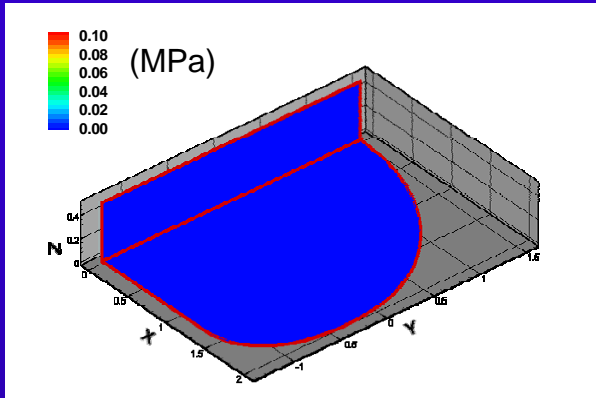


Homogeneous

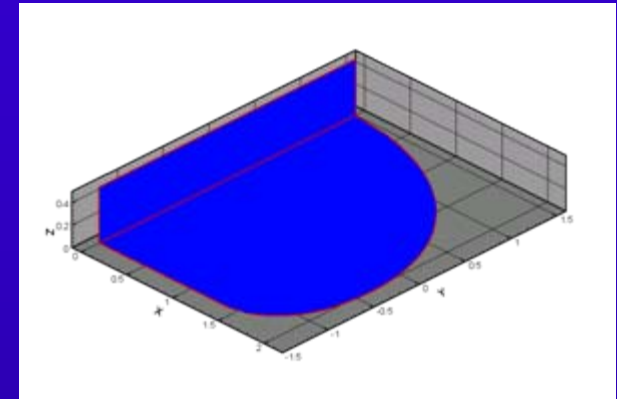
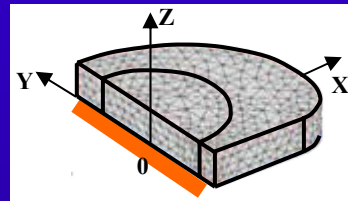
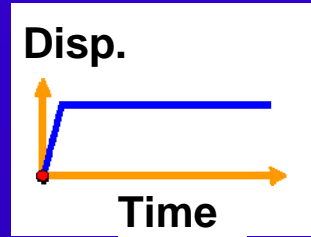
Inhomogeneous



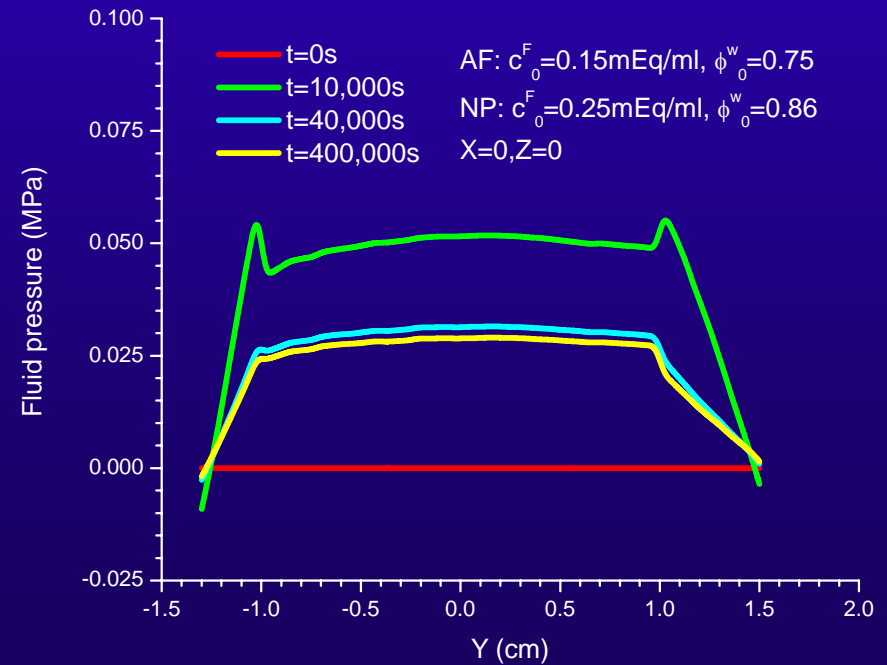
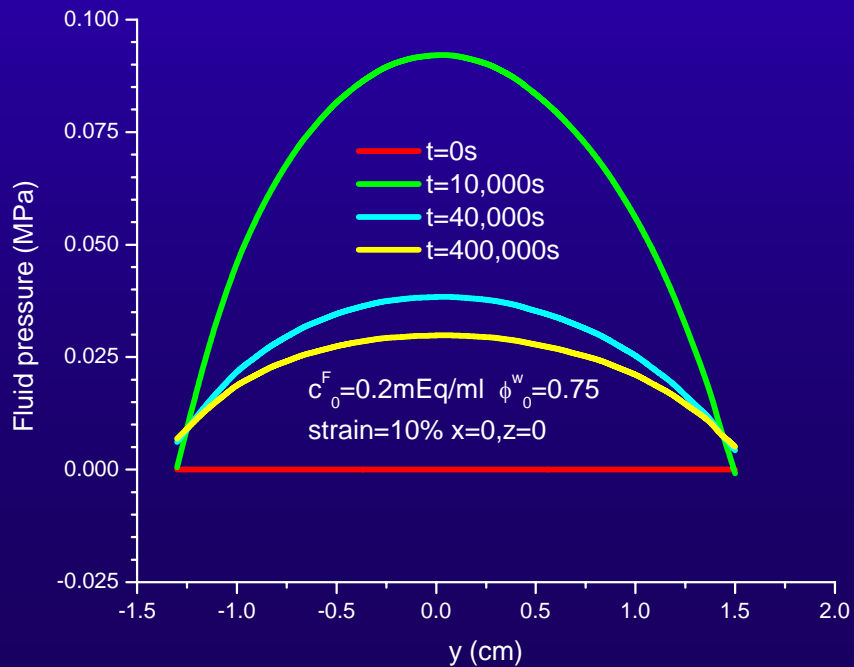
Fluid Pressure



Homogeneous

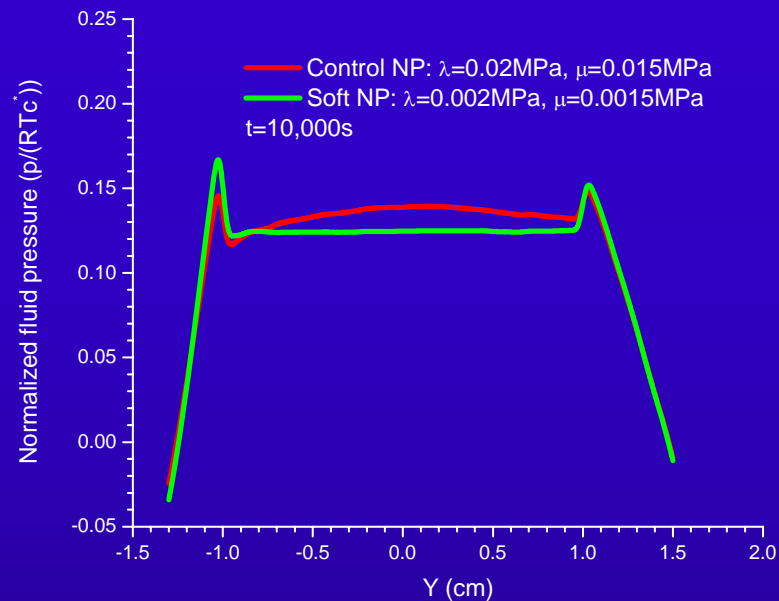
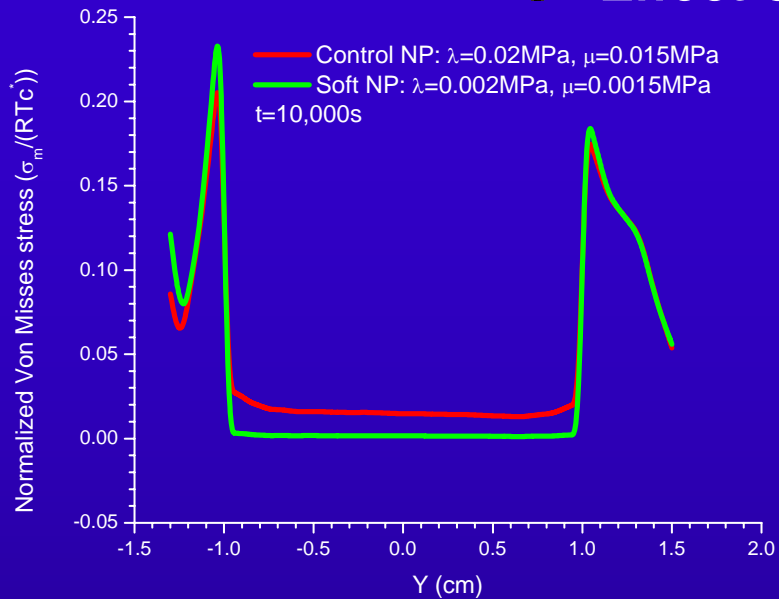


Inhomogeneous

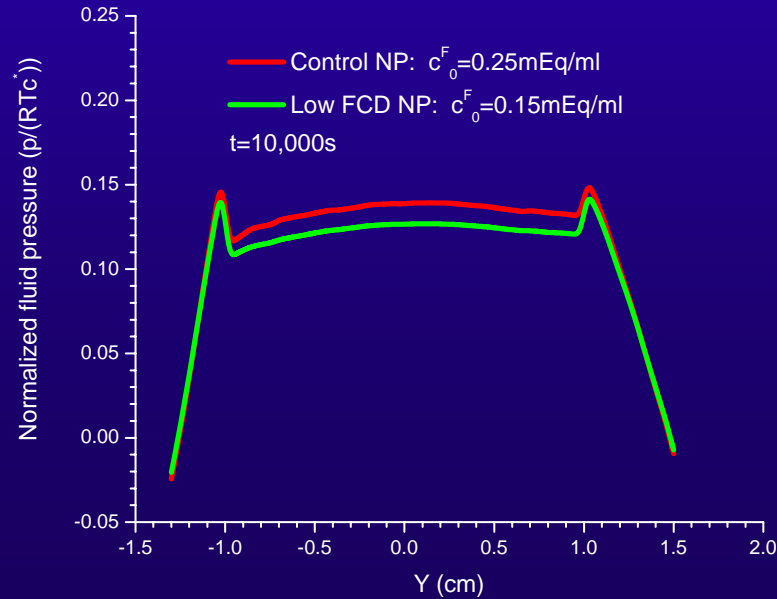
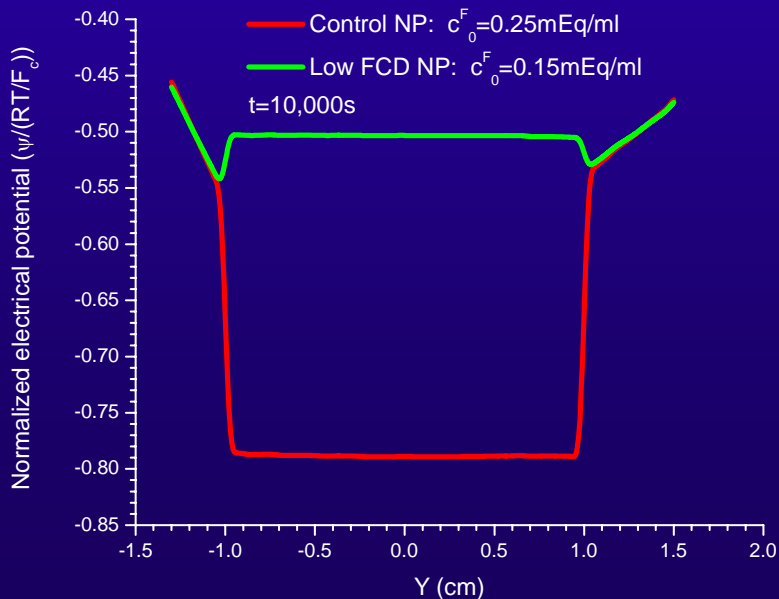




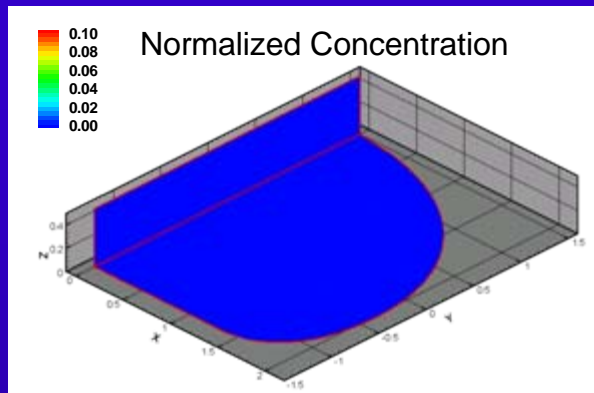
Effect of elastic modulus of NP



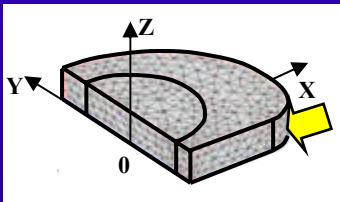
Effect of FCD of NP



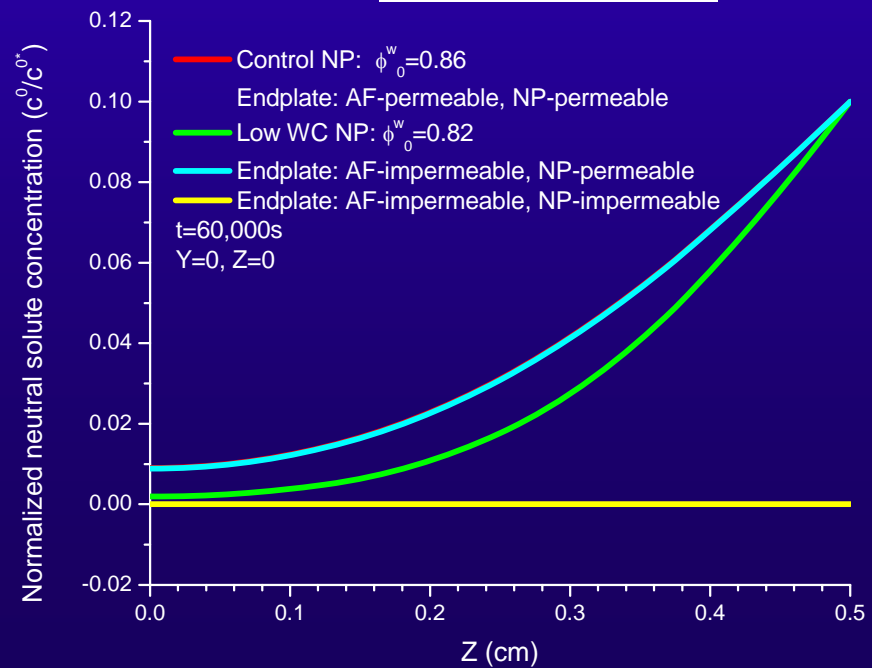
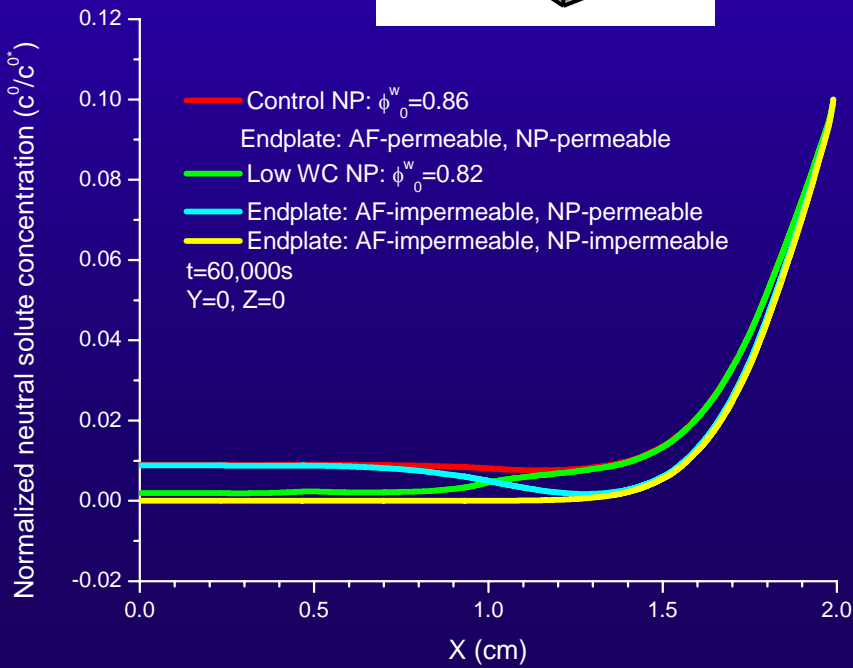
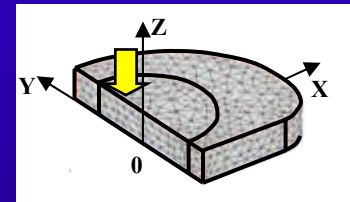
Neutral Solute Transport



Large solute



Axial direction



Discussion

- Inhomogeneous material properties have significant effect on the distribution of physical signals within the IVD
- The change of tissue composition (e.g., water content and FCD) significantly affect physical signals (e.g., electrical potential and fluid pressure)
- Electrical signal is an sensitive indicator for tissue degeneration
- The permeability of endplate affects solute transport into the tissue

Future Study

● Improve 3D finite element model

- Anisotropy
- Tension-compression nonlinearity
- Effect of solute binding to matrix
- Endplate
- Chemical reactions (production and consumption)

● Model validation

Acknowledgement



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